WJEC 2014 Online Exam Review

GCE Mathematics C3 0975-01

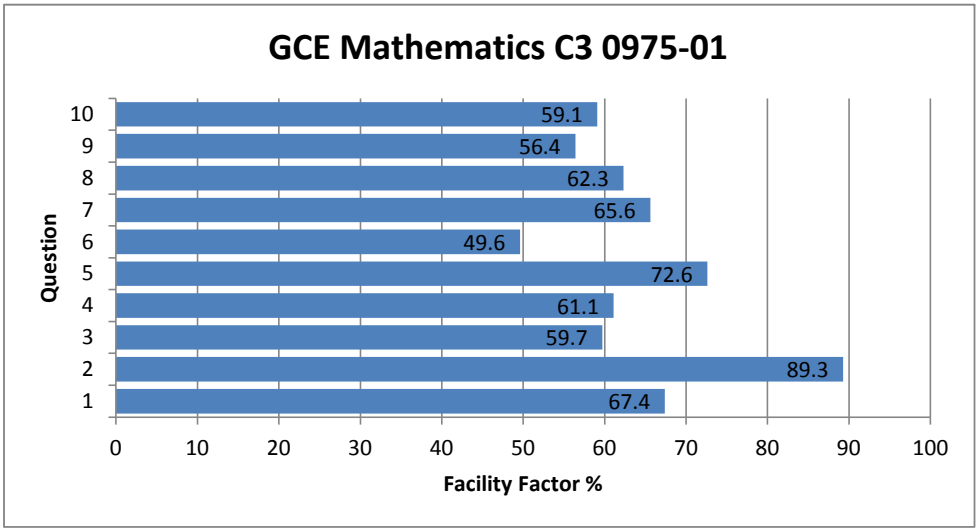
All Candidates' performance across questions

Question Title	N	Mean	S D	Max Mark	F F	Attempt %
1	2240	4	0.9	6	67.4	99.9
2	2234	5.4	1.5	6	89.3	99.6
3	2233	4.8	1.5	8	59.7	99.6
4	2223	4.3	2.1	7	61.1	99.2
5	2224	6.5	2.6	9	72.6	99.2
6	2232	5.5	2.7	11	49.6	99.5
7	2214	6.6	2.6	10	65.6	98.8
8	2218	3.1	1.7	5	62.3	98.9
9	2194	2.8	1.7	5	56.4	97.9
10	2173	4.7	2.4	8	59.1	96.9

1b

3b

4



1. *(b)* **Use your answer to part *(a)*** to deduce an approximate value for the integral

$$\int_0^3 \ln\left(16 + 2e^x\right) dx.$$

[2]

1. b) $\int_0^3 \ln(16 + 2e^x)\,dx$

$2 \times$ [illegible] ≈ 7.82

$= 15.64$

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$2 \times$ ~~$\int_0^3$~~ ~~$\ln$~~ 7.82

$= 15.64$ X

M0

A0

(4)

1. 6. $\int_0^3 \ln(16+2e^x)\,dx \approx \ln 2 + \int_0^3 \ln(8+e^x)\,dx$

$\approx \ln 2 + 7.820968183$

≈ 8.514115366

≈ 8.51 (2.d.p)

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1.b) $\int_0^3 \ln(16+2e^x)\,dx = \int_0^3 \ln(2)+\ln(8+e^x)\,dx$

$\approx 7.82 \times \ln(2)$

≈ 5.42

1. b)	$\int_0^3 \ln(16+2e^x)\,dx = \int_0^3 \ln(2)+\ln(8+e^x)\,dx$	M1
	$\approx 7.82 \times \ln(2)$	A0
	≈ 5.42	

1. (b) $\int_0^3 \ln(16+2e^x)\,dx = \int_0^3 \ln(2(8+e^x))\,dx$

$= \int_0^3 \ln(2) + \ln(8+e^x)\,dx$

$= \ln(2) + 7.82$

$= 8.51$ (2dp's)

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$= \ln(2) + 7.82$

$= 8.51$ (2dp's)

3. *(b)* Show that there is no point on *C* at which $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. [4]

3. b) $\frac{x-2y^2}{y^3+4xy} = 0$

either $x-2y^2=0$ or $y^3+4xy=0$

$x-2y^2=0$

$x=2y^2$

sub $2y^2$ into $y^3+4xy=0$

$y^3+4\times 2y^2\times y=0$

$y^3+8y^3=0$

$9y^3=0$

$y^3=0$

y^3 cannot $=0$

∴ there is no point on C at which $\frac{dy}{dx}=0$

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$$\frac{x - 2y^2}{y^2 + 4xy} = 0$$

$$\frac{2}{y + 4y} = 0$$

$$\frac{2}{5y} = 0$$

$$2 = 5y$$

$$y = 2.5$$

$$x - 2y^2 = y^3 + 4xy$$

$$x - 4xy = y^3 + 2y^2$$

$$x(1 - 4y) = y^3 + 2y^2$$

$$x = \frac{y^3 + 2y^2}{1 - 4y}$$

$$x = \frac{y^2 + 2y}{-3}$$

$$x - 2y^2 = y^3 + 4xy$$

$$x - 4xy = y^3 - 2y^2$$

$$x(1 - 4y) = y^2(y - 2)$$

$$x = y^2(y - 2)$$

$$\frac{x(1 - 4y)}{(y - 2)} = y^2$$

$$y = \sqrt{\frac{x(1 - 4y)}{y - 2}}$$

3. b. $\frac{dy}{dx} = 0$ $\quad \frac{x - 2y^2}{y^3 + 4xy} = 0$

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$\frac{2}{5y} = 0$

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$x = y^2(y - 2)$

$\frac{x(1 - 4y)}{(y - 2)} = y^2$

$y = \sqrt{\frac{x(1 - 4y)}{y - 2}}$

4. Given that $x = 2e^t - 5$, $y = 8e^{-t} + 3e^t - 4$, find the value of t when $\frac{dy}{dx} = -1$.

Give your answer correct to three decimal places. [7]

4. $\frac{dy}{dx} = \frac{-8e^{-t} + 3e^{t}}{2e^{t}}$

$\frac{-8e^{-t} + 3e^{t}}{2e^{t}} = -1$

$-8e^{-t} + 3e^{t} = -2e^{t}$

$-8e^{-t} = -5e^{t}$

$8e^{-t} = 5e^{t}$

$\frac{dy}{dx} = \frac{-8e^{-t}}{2e^{t}} + \frac{3e^{t}}{2e^{t}}$

$= \frac{-8e^{-t}}{2e^{t}} + \frac{3}{2}$

$= -4e^{-t^2} + \frac{3}{2}$

$-4e^{-t^2} + \frac{3}{2} = -1$

$-4e^{-t^2} = -\frac{5}{2}$

$4e^{-t^2} = \frac{5}{2}$

$e^{-t^2} = \frac{5}{8}$

$-t^2 = \ln\left(\frac{5}{8}\right)$

$-t = \pm\sqrt{\ln\left(\frac{5}{8}\right)}$

$-t^2 = -0.470036292$

$t^2 = 0.470036292$

$t = 0.686$ (3dp)

4.

$$\frac{dy}{dx} = \frac{-8e^{-t} + 3e^{t}}{2e^{t}}$$ **M1 A1**

$$\frac{-8e^{-t} + 3e^{t}}{2e^{t}} = -1$$

$$-8e^{-t} + 3e^{t} = -2e^{t}$$

$$-8e^{-t} = -5e^{t}$$

$$8e^{-t} = 5e^{t}$$

$$\frac{dy}{dx} = \frac{-8e^{-t}}{2e^{t}} + \frac{3e^{t}}{2e^{t}}$$

$$= \frac{-8e^{-t}}{2e^{t}} + \frac{3}{2}$$ **M0**

$$= -4e^{-t^2} + \frac{3}{2}$$

$$-4e^{-t^2} + \frac{3}{2} = -1$$ **A0**

$$-4e^{-t^2} = -\frac{5}{2}$$

$$4e^{-t^2} = \frac{5}{2}$$

$$e^{-t^2} = \frac{5}{8}$$

$$-t^2 = \ln\left(\frac{5}{8}\right)$$ **A0**

$$-t = \pm\sqrt{\ln\left(\frac{5}{8}\right)}$$

$$-t^2 = -0.470036292$$

$$t^2 = 0.470036292$$

$$t = 0.686 \quad (3\text{dp})$$

4. $x = 2e^t - 5$ $\quad y = 8e^{-t} + 3e^t - 4$

$\frac{dx}{dt} = 2e^t$ $\quad \frac{dy}{dt} = -8e^{-t} + 3e^t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-8e^{-t} + 3e^t}{2e^t}$$

when $\frac{dy}{dx} = -1$.

$$\frac{-8e^{-t} + 3e^t}{2e^t} = -1$$

$$-8e^{-t} + 3e^t = -2e^t$$

$$8t + 3t = -2t$$

$$t = 0$$

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when $\frac{dy}{dx} = -1$.

$\frac{-8e^{-t} + 3e^t}{2e^t} = -1$

$-8e^{-t} + 3e^t = -2e^t$

$8t + 3t = -2t$.

$t = 0$

4.

$$\frac{dy}{dx} = -1$$

$$\frac{-8e^{-t} + 3e^{t}}{2e^{t}} = -1$$

$$-8e^{-t} + 3e^{t} = -2e^{t}$$

$$-8e^{-t} + 3e^{t} + 2e^{t} = 0$$

$$-8e^{-t} + 5e^{t} = 0$$

$$-8e^{-t} = -5e^{t} \quad \text{(Ln both sides)}$$

$$\ln(-8e^{-t}) = \ln(-5e^{t})$$

$$-8(-t) = -5(t)$$

$$8t = -5t$$

$$8t + 5t = 0$$

$$13t = 0 \quad (?)$$

4.	$\frac{dy}{dx} = -1$	
	$\frac{-8e^{-t} + 3e^{t}}{2e^{t}} = -1$	
	$-8e^{-t} + 3e^{t} = -2e^{t}$	**M1**
	$-8e^{-t} + 3e^{t} + 2e^{t} = 0$	
	$-8e^{-t} + 5e^{t} = 0$	
	$-8e^{-t} = -5e^{t}$ (Ln both sides)	
	$\ln(-8e^{-t}) = \ln(-5e^{t})$	**A0**
	$-8(-t) = -5(t)$	
	$8t = -5t$	
	$8t + 5t = 0$	**A0**
	$13t = 0$ (?)	